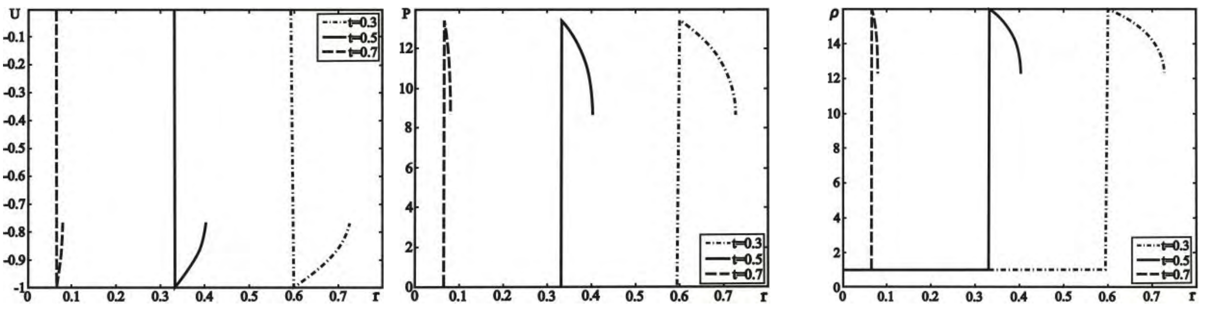

Рисунок 3. Зависимости скорости, давления и плотности от эйлеровой координаты r между ударной волной и характеристикой для γ=5/3, n=3 больше n_* =2,065135 и трех моментов времени t=0,3; 0,5; 0,7

В лагранжевых координатах построено аналитическое решение задачи о сходящейся ударной волне для произвольных показателей n, которые определяют схождение ударной волны.

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THE ANALYTICAL SOLUTION OF THE PROBLEM OF A CONVERGENT SHOCK WAVE FOR ARBITRARY COEFFICIENTS OF SELF-SIMILARITY

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Is being considered the gas sphere with the mass M_0 and initial parameters for the gas ρ_0=const, U_0=0, P_0=0, E_0=0, where ρ – density, U – velocity, P – pressure, E – specific internal energy at $t = t_0$. The problem has spherical symmetry. The Lagrangian coordinate is a spherical mass M. The second independent variable is the time t. At the point t_0, M_0 the velocity is set $U_1 < 0$. When $t > t_0$ the shock wave spreads from this point into the center of symmetry and it focuses to the point $M = 0$ at the moment t_f. At $t > t_0$ the sphere boundary moves in the variables r, t, but it is a vertical line in variables M, t. Generally speaking, all the trajectories of the particles are vertical lines. The value of entropy which appeared on the shock wave retained along each of the trajectories. Parameters of the gas between the shock wave and the boundary are determined by the system of Euler-Helmholtz conservation laws. Take the equation of state for ideal gas in two forms

$$P=(\gamma-1)\rho E,\quad P=F(s)\rho^{\gamma}, \tag{1}$$

where $F(s)$ – entropy function.

Conservation laws on the shock wave at $U_0=0,\ P_0=0,\ E_0=0,\ F_0=0$ are of the form [1]

$$\rho_w(D-U_w)-\rho_0 D=0, \tag{2}$$

$$\rho_0 D U_w - P_w=0, \tag{3}$$

$$\rho_0 D\left(E_w+\frac{1}{2}U_w^2\right)-P_w U_w=0. \tag{4}$$

The index "w" indicats values on the shock wave, D – the shock velocity. We transform these equations to a form containing the dependence $U_w,\ \rho_w,\ F_w,\ P_w$ on the velocity of the shock wave in Lagrangian coordinates. The Lagrangian coordinate M_w of the shock wave in the case of a spherically symmetric flow associated by the equation with its coordinate Euler r_w

$$M_w=\frac{4}{3}\pi\rho_0 r_w^3. \tag{5}$$

The velocity of a shock wave in Lagrangian coordinates is a change M_w in course of time

$$W=\frac{dM_w}{dt}=4\pi\rho_0 r_w^2 D. \tag{6}$$

We replace the Euler coordinate of the shock wave by its Lagrangian coordinate. For this we express r_w from (5) and substitute in (6)

$$W=(3M_w)^{2/3}(4\pi\rho_0)^{1/3}\,D. \tag{7}$$

Expressing in (7) D through W and M_w and substituting in (2)–(4) with the help of (1) we obtain the expressions

$$\rho_w=\frac{\gamma+1}{\gamma-1}\rho_0. \tag{8}$$

$$U_w=\frac{2}{\gamma+1}(4\pi\rho_0)^{-1/3}(3M_w)^{-2/3}\,W. \tag{9}$$

$$P_w=\frac{2}{\gamma+1}\rho_0^{1/3}(4\pi)^{-2/3}(3M_w)^{-4/3}\,W^2. \tag{10}$$

From (1), (8) and (10) ensue expression for F_w

$$F_w=\left[\frac{2}{\gamma+1}\left(\frac{\gamma-1}{\gamma+1}\right)^{\gamma}\rho_0^{-(\gamma-1/3)}(4\pi)^{-2/3}\right](3M_w)^{-4/3}\,W^2. \tag{11}$$

At the point $t=t_0\quad M_w=M_0, U_w=U_{w0}, P_w=P_{w0}, F_w=F_{w0}$.

$$U_w=U_0\left(\frac{W}{W_0}\right)\left(\frac{M_0}{M_w}\right)^{2/3},\quad P_w=P_0\left(\frac{W}{W_0}\right)^2\left(\frac{M_0}{M_w}\right)^{4/3},\quad F_w=F_0\left(\frac{W}{W_0}\right)^2\left(\frac{M_0}{M_w}\right)^{4/3}. \tag{12}$$

We define the trajectory of the shock wave by analogy with [2, 3]

$$M_w=M_0\,\varphi(t)^n, \tag{13}$$

where $\varphi=(t_f-t)/(t_f-t_0)$. Differentiating M_w on t, we obtain an expression for the shock wave velocity in Lagrangian coordinates

$$W=W_0\,\varphi^{n-1}, \tag{14}$$

where

$$W_0=-\frac{M_0 n}{t_f-t_0}. \tag{15}$$

Excluding the functions of the time in (13) and (14), we obtain the dependence W on M_w

$$\frac{W}{W_0} = \left(\frac{M_w}{M_0}\right)^{(n-1)/n}. \tag{16}$$

With the help of (16) excluding W in (12)

$$U_w = U_{w0}\left(\frac{M_w}{M_0}\right)^{(n-3)/3n}, \quad P_w = P_{w0}\left(\frac{M_w}{M_0}\right)^{2(n-3)/3n}, \quad F_w = F_{w0}\left(\frac{M_w}{M_0}\right)^{2(n-3)/3n}. \tag{17}$$

The value F_w along the trajectory of particle with coordinate M_w is constant. Consequently, the dependence of the entropy on the mass between the shock wave and the boundary of the gas has the form

$$F = F_{w0}\left(\frac{M}{M_0}\right)^{2(n-3)/3n}. \tag{18}$$

From (5) ensue expression r_w from M_w

$$r_w = r_0\left(\frac{M_w}{M_0}\right)^{1/3}. \tag{19}$$

Parameters of the adiabatic flow between the shock wave and boundary of the gas are determined by the equations of the trajectory, the conservation of mass and motion

$$\left(\frac{\partial r}{\partial t}\right)_M - U = 0, \tag{20}$$

$$\left(\frac{\partial \rho}{\partial t}\right)_M + 4\pi\rho^2 \frac{\partial\left(r^2 U\right)}{\partial M} = 0, \tag{21}$$

$$\left(\frac{\partial U}{\partial t}\right)_M + 4\pi r^2 \frac{\partial\left(F\rho^\gamma\right)}{\partial M} = 0. \tag{22}$$

These equations contain three desired functions r, ρ and U. The value F is determined on the shock wave and depends only on M (17).

Let us pass in (20)–(22) to new desired functions

$$R = r^3, \quad C = r^2 U. \tag{23}$$

After the pass to the functions R and C the equations (20)–(22) take the form

$$\left(\frac{\partial R}{\partial t}\right)_M - 3C = 0, \tag{24}$$

$$\left(\frac{\partial \rho}{\partial t}\right)_M + 4\pi\rho^2 \frac{\partial C}{\partial M} = 0, \tag{25}$$

$$\left(\frac{\partial C}{\partial t}\right)_M + 4\pi R^{4/3} \frac{\partial\left(F\rho^\gamma\right)}{\partial M} - 2C^2 R^{-1} = 0. \tag{26}$$

From (17), (19) and (23) follow the dependence of R_w and C_w on M_w

$$R_w = R_0 \frac{M_w}{M_0}, \quad C_w = C_0\left(\frac{M_w}{M_0}\right)^{(n-1)/n}. \tag{27}$$

The equations (24)–(26) are essential for finding R, C and ρ ρ in the area of the integration $M_w \le M \le M_0$, $t_0 \le t \le t_f$.

Let us proceed from the variables t, M to variables t, $\xi(t, M)$. The equations (24)–(26) take the forms

$$\left(\frac{\partial R}{\partial t}\right)_\xi + \left(\frac{\partial R}{\partial \xi}\right)_t \left(\frac{\partial \xi}{\partial t}\right)_M - 3C = 0, \tag{28}$$

$$\left(\frac{\partial \rho}{\partial t}\right)_\xi + \left(\frac{\partial \rho}{\partial \xi}\right)_t \left(\frac{\partial \xi}{\partial t}\right)_M + 4\pi\rho^2 \left(\frac{\partial C}{\partial \xi}\right)_t \left(\frac{\partial \xi}{\partial M}\right)_t = 0, \tag{29}$$

$$\left(\frac{\partial C}{\partial t}\right)_\xi+\left(\frac{\partial C}{\partial \xi}\right)_t\left(\frac{\partial \xi}{\partial t}\right)_M-\frac{2C^2}{R}+ \\ +4\pi R^{4/3}\left[\rho^\gamma\left(\frac{\partial F}{\partial \xi}\right)_t\left(\frac{\partial \xi}{\partial M}\right)_t+\gamma F\rho^{\gamma-1}\left(\frac{\partial \rho}{\partial \xi}\right)_t\left(\frac{\partial \xi}{\partial M}\right)_t\right]=0. \tag{30}$$

We define the dependence of $\xi(t,\,M)$ so that on the shock wave will be $\xi=1$. From (13) follow that it's easiest to take such a dependence in the form

$$\xi=\frac{M}{M_0}\varphi^{-n}. \tag{31}$$

To separate the variables representing $R,\,\rho$ and C as functions of time multiplied by functions of ξ

$$R=\alpha_R(t)T(\xi)\qquad \rho=\alpha_\rho(t)\delta(\xi)\qquad C=\alpha_C(t)Z(\xi). \tag{32}$$

Since $\xi=1$ on the shock wave, the values $T_1=T(1)$, $\delta_1=\delta(1)$, $Z_1=Z(1)$ must be constant. The dependence $R_w(t)$ is obtained from (27) and (31)

$$R_w=R_0\,\varphi^n. \tag{33}$$

Comparing this dependence with (32) on the shock wave, we obtain an expression for α_R

$$\alpha_R(t)=R_0\,\varphi^n\,T_1^{-1}. \tag{34}$$

Similarly for α_ρ and α_C we obtain the relations

$$\alpha_\rho=\rho_0\left(\frac{\gamma+1}{\gamma-1}\right)\delta_1^{-1},\qquad \alpha_C(t)=C_0\,\varphi^{n-1}\,Z_1^{-1}. \tag{35}$$

By substituting (32)–(35) into (28)–(30) and taking (15), we obtain three equations for $T,\,\delta$ and Z

$$\xi T'=A_1, \tag{36}$$

$$\delta_1 B_1 Z'-\xi Z_1\delta'=0, \tag{37}$$

$$-\frac{\xi}{Z_1}Z'+\frac{C_1\gamma\xi}{\delta_1}\delta'=C_2, \tag{38}$$

where the mark indicates differentiation of ξ. The coefficients of the equations (36)–(38) A_1, B_1, C_1, C_2 with the help of (5), (6), (12) and (21) take the forms

$$A_1=T-\frac{2ZT_1}{(\gamma+1)Z_1},\quad B_1=\frac{2\delta^2}{(\gamma-1)\delta_1^2},\quad C_1=\frac{\delta^{\gamma-1}\,T^{4/3}\,\xi^{-(n+6)/3n}}{\delta_1^{\gamma-1}\,T_1^{4/3}}, \\ C_2=\frac{4Z^2\,T_1}{3(\gamma+1)Z_1^2\,T}-\frac{(n-1)Z}{nZ_1}-C_1\frac{2(n-3)\delta}{3n\,\delta_1}. \tag{39}$$

For T',δ',Z' these equations (36)-(38) give a system of linear homogeneous equations. he determinant of the system is the following

$$\Delta=B_1C_1\gamma\xi-\xi^2.$$

If $\Delta\neq 0$, the solution of the system (36)–(38) exists and has the form

$$T'=\frac{A_1}{\xi},\quad \delta'=\frac{B_1\,C_2\,\delta_1}{\Delta},\quad Z'=\frac{\xi C_2\,Z_1}{\Delta}. \tag{40}$$

On the shock wave at $\xi=1$ the functions T, δ, Z, Δ and coefficients (39) take values

$$T=T_1,\quad \delta=\delta_1,\quad Z=Z_1,\quad A_1=\frac{(\gamma-1)T_1}{\gamma+1},\quad B_1=\frac{2}{(\gamma-1)}, \\ C_1=1,\quad C_2=\frac{9(\gamma+1)-n(5\gamma+1)}{3n(\gamma+1)},\quad \Delta(1)=\frac{(\gamma+1)}{(\gamma-1)}.$$

Integrating the system of the equations (40) begins at the point ξ=1 (on the shock wave). Calculations show that there exist interval values of n such that the determinant does not vanish. At some value of n_*

the determinant vanishes at $\xi = \xi_*$. At this point, the solution exists if C_2 is vanishes also. Each value γ corresponds to a unique value n_* (Table 1). In this table shows the values ξ_*, in which simultaneously $\Delta(\xi_*) = 0,\ C_2(\xi_*) = 0$. Profiles of velocities, pressures and densities are shown in Figure 1 for three time points at $n = n_*$.

Table 1. The values of n_* and ξ_* corresponding to different values of the adiabatic coefficient γ

γ	1,1	1,2	4/3	1,4	5/3
n_*	2.387916	2.271434	2.183068	2.151532	2.065135
ξ_*	7.959997	5.717071	4.559431	4.227062	3.481885

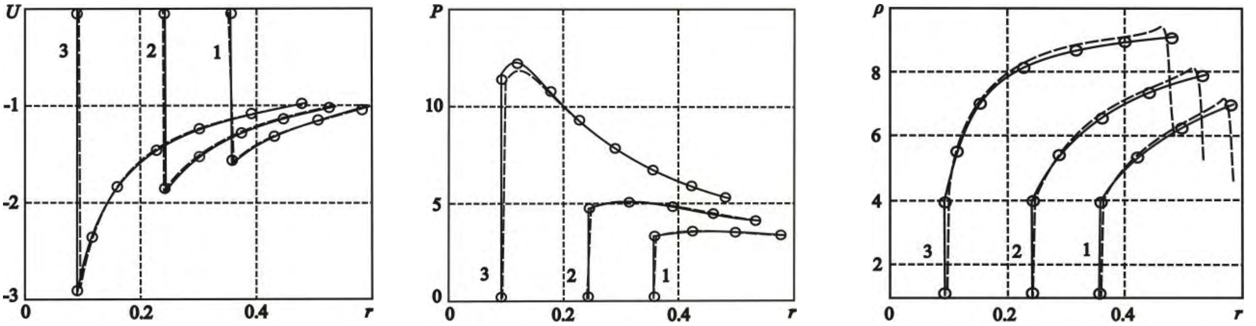

Figure 1. Dependences of velocity, pressure, and density on the Euler coordinate r for γ=5/3, $n = n_* = 2,065135$ and three time points 1 – t=0,4; 2 – t=0,45; 3 – t= 0,5. The solid lines – the analytical solution derived in this work, –o– – calculations by the VOLNA code [4] with shock smearing, ----- – calculations by the VOLNA code with no shock smearing

When $0 < n < n_*$ the determinant of the system (40) is positive in the whole period of change ξ $1 \le \xi < \infty$. In this case, there is a collapse of the gas sphere - its volume approaches to zero. The structure of the gas flow between the shock wave front and boundary of the sphere is shown in Figure 2.

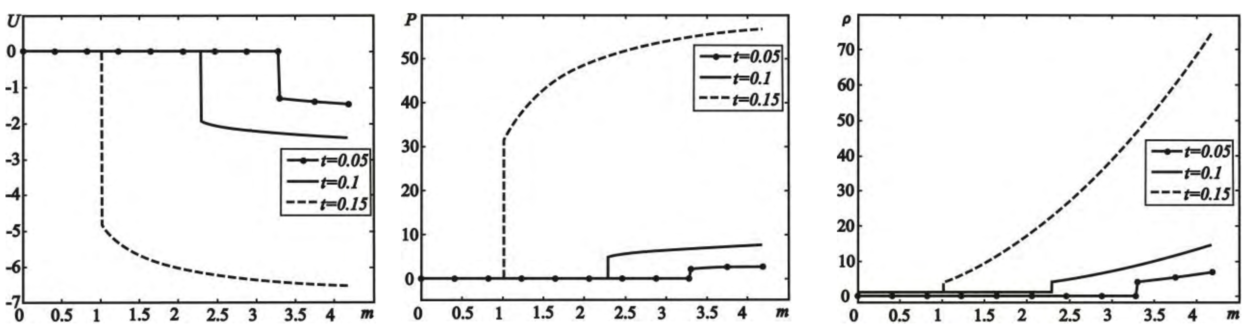

Figure 2. Dependencies of velocity, pressure, and density on the Lagrangian coordinate for γ=5/3, n=0,68 less n_*=2,065135 and three time points t=0,05; 0,1; 0,15

In the area $n > n_{*\ *}$ the determinant vanishes for some value ξ_n, which is depends on n. But at this point $C_2(\xi_n)$ does not vanish. Thus, the solution exists in the area $1 \le \xi > \xi_n$. On the border of the gas sphere at $M = M_0$ the value of ξ_n is reached at the moment

$$t_n = t_f - \left(t_f - t_0\right)\xi_n^{-1/n}.$$

From the point M_0, t_n comes out the line, on which $\xi = \xi_n$

$$M_n = M_0 \xi_n \left(\frac{t_f - t}{t_f - t_0}\right)^n. \tag{41}$$

This is a characteristic. This line is focuses simultaneously with the shock wave, because at $t = t_f$, $M_n = 0$. In the area between (41) and the shock wave (13), for each $n > n_*$ the single solution is exists. The structure of the gas flow is shown in Figure 3.

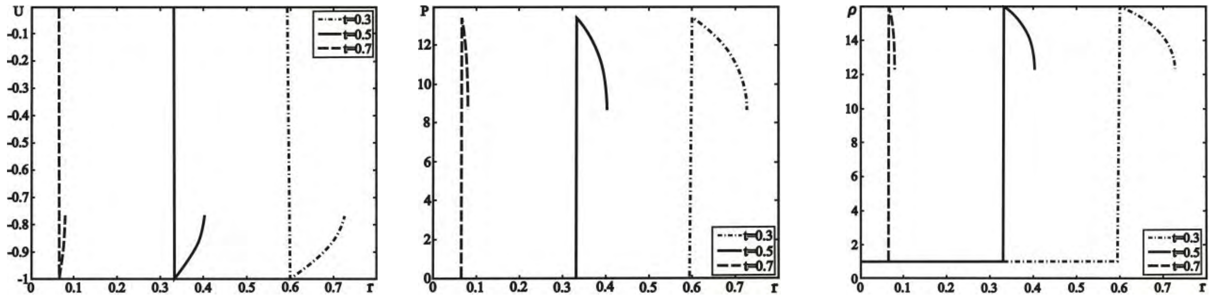

Figure 3. Dependencies of velocity, pressure, and density on the Euler coordinate r between the shock wave and characteristic for γ=5/3, n=3 larger n_*=2,065135 and three time points t=0,3; 0,5; 0,7

The analytical solution of the problem of a converging shock wave in the collapsing gas has been constructed in Lagrangian coordinates with arbitrary parameter n, which determines the convergence of the shock wave.

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ОБ ИНЕРЦИОННОМ РАСТЯЖЕНИИ МЕТАЛЛИЧЕСКИХ КУМУЛЯТИВНЫХ СТРУЙ В ПРОДОЛЬНОМ МАГНИТНОМ ПОЛЕ

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Как свидетельствуют эксперименты [1], предварительное создание аксиального магнитного поля в металлической облицовке кумулятивного заряда непосредственно перед его подрывом может существенно влиять на функционирование заряда. В опытах с кумулятивными зарядами диаметром d_0 = 50 мм, имеющими медную коническую облицовку, при индукции начального поля в облицовке B_{l0} в десятые доли тесла наблюдалось значительное снижение пробивного действия заряда. Теоретический анализ данного эффекта [2–4] позволяет предположить главной причиной его проявления резкое усиление магнитного поля в области образования кумулятивной струи (КС) при схлопывании «намагниченной» облицовки. Генерирование сильного магнитного поля в